

Example 4 Rationalize the denominator of $\frac{3}{5 - 2\sqrt{7}}$.

Solution

$$\begin{aligned}\frac{3}{5 - 2\sqrt{7}} &= \frac{3}{5 - 2\sqrt{7}} \cdot \frac{5 + 2\sqrt{7}}{5 + 2\sqrt{7}} \\ &= \frac{3(5 + 2\sqrt{7})}{25 - (2\sqrt{7})^2} \\ &= \frac{15 + 6\sqrt{7}}{25 - 28} \\ &= \frac{15 + 6\sqrt{7}}{-3} \\ &= \frac{15}{-3} + \frac{6\sqrt{7}}{-3} \\ &= -5 - 2\sqrt{7} \quad \text{Answer}\end{aligned}$$

Oral Exercises

Complete. Express in simplest form.

- $(\sqrt{5} + 3)(\sqrt{5} - 3) = 5 - \underline{\quad} = \underline{\quad}$
- $(8 - \sqrt{6})(8 + \sqrt{6}) = 64 - \underline{\quad} = \underline{\quad}$
- $(\sqrt{2} + 3)^2 = 2 + \underline{\quad} + 9 = 11 + \underline{\quad}$
- $(6 - \sqrt{11})^2 = 36 - \underline{\quad} + \underline{\quad} = \underline{\quad}$

State the conjugate of each binomial.

- $8 + 4\sqrt{5}$
- $-7 - 3\sqrt{11}$
- $-4 + 6\sqrt{7}$
- $5 - 8\sqrt{13}$

Written Exercises

Simplify.

- A**
- $(2 + \sqrt{3})(2 - \sqrt{3})$
 - $(4 + \sqrt{11})(4 - \sqrt{11})$
 - $(\sqrt{15} + 6)(\sqrt{15} - 6)$
 - $(\sqrt{19} - 9)(\sqrt{19} + 9)$
 - $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$
 - $(\sqrt{15} - \sqrt{3})(\sqrt{15} + \sqrt{3})$
 - $(2 + \sqrt{7})^2$
 - $(8 - \sqrt{6})^2$
 - $(2\sqrt{2} - 3)^2$
 - $(4\sqrt{10} + 3)^2$
 - $(\sqrt{13} - 2\sqrt{5})^2$
 - $(3\sqrt{7} - \sqrt{3})^2$
 - $(2\sqrt{7} + \sqrt{3})(2\sqrt{7} - \sqrt{3})$
 - $(3\sqrt{5} - \sqrt{2})(3\sqrt{5} + \sqrt{2})$
 - $(6\sqrt{5} - \sqrt{7})(6\sqrt{5} + \sqrt{7})$
 - $(8\sqrt{11} + 2\sqrt{6})(8\sqrt{11} - 2\sqrt{6})$
 - $(4\sqrt{3} - 5)(2\sqrt{3} + 3)$
 - $(6\sqrt{2} + 4)(3\sqrt{2} - 5)$
- B**
- $(2\sqrt{5} - 6\sqrt{7})(3\sqrt{5} + \sqrt{7})$
 - $(7\sqrt{13} + 2\sqrt{6})(2\sqrt{13} + 3\sqrt{6})$
 - $(4\sqrt{11} - 2\sqrt{2})(6\sqrt{11} + 8\sqrt{2})$
 - $(8\sqrt{6} - 2\sqrt{3})(2\sqrt{6} - 3\sqrt{3})$

Rationalize the denominator of each fraction.

23. $\frac{1}{1 + \sqrt{5}}$

24. $\frac{1}{2 + \sqrt{3}}$

25. $\frac{3}{\sqrt{5} - 2}$

26. $\frac{1}{\sqrt{7} - 3}$

27. $\frac{1 + \sqrt{7}}{2 - \sqrt{7}}$

28. $\frac{4 + \sqrt{5}}{3 - \sqrt{5}}$

29. $\frac{\sqrt{3} - 4}{\sqrt{7} + 2}$

30. $\frac{\sqrt{5} - 2}{\sqrt{3} + 1}$

31. $\frac{7}{2\sqrt{5} + 1}$

32. $\frac{5}{3\sqrt{7} - 5}$

33. $\frac{4 + 2\sqrt{2}}{2\sqrt{5} - 3}$

34. $\frac{6 - 2\sqrt{3}}{3\sqrt{2} + 3}$

If $f(x) = x^2 - 5x - 7$, find the value of each function.

Sample $f(\sqrt{7}) = (\sqrt{7})^2 - 5(\sqrt{7}) - 7$
 $= 7 - 5\sqrt{7} - 7$
 $= -5\sqrt{7}$ *Answer*

35. $f(\sqrt{6})$

36. $f(\sqrt{10})$

37. $f(\sqrt{2} + 1)$

38. $f(\sqrt{3} + 2)$

39. $f(-2 + \sqrt{11})$

40. $f(\sqrt{7} - 2)$

41. Show that $(4 + \sqrt{7})$ and $(4 - \sqrt{7})$ are roots of the equation $x^2 - 8x + 9 = 0$.
 42. Show that $(5 + \sqrt{3})$ and $(5 - \sqrt{3})$ are roots of the equation $y^2 - 10y + 22 = 0$.
 43. Show that $\left(\frac{2}{3} + \frac{\sqrt{7}}{3}\right)$ and $\left(\frac{2}{3} - \frac{\sqrt{7}}{3}\right)$ are roots of the equation $3x^2 - 4x - 1 = 0$.

- C** 44. Write an expression in simplest form for the area of a triangle whose base is $\frac{4\sqrt{5} - 2}{5}$ units and whose height is $\frac{\sqrt{5} + 6}{2}$ units.

Simplify each expression, assuming that the value of each variable is nonnegative.

45. $(x + \sqrt{y})(x - \sqrt{y})$

46. $(x - 3\sqrt{2})^2$

47. $(3a\sqrt{b} - c)(5a\sqrt{b} + 3c)$

Mixed Review Exercises

Simplify. Assume the radicands are nonnegative real numbers.

1. $\sqrt{18x^8}$

2. $4\sqrt{15x} \cdot 3\sqrt{5}$

3. $6\sqrt{8} - 4\sqrt{2}$

4. $4\sqrt{63} + 5\sqrt{28}$

5. $\sqrt{\frac{3}{5}} \cdot \sqrt{\frac{5}{9}}$

6. $\sqrt{1\frac{5}{6}} \cdot \sqrt{4\frac{1}{6}}$

7. $(3 - 5k^2)^2$

8. $(2p + 7z)^2$

9. $(6ab + x)(6ab - x)$

Solve.

10. $7p - 3 = 6(p + 2)$

11. $x^2 - 14x + 45 = 0$

12. $36g^2 = 16$